4.1 Apply Triangle Sum Properties

Before You classified angles and found their measures.

Now You will classify triangles and find measures of their angles.

Why? So you can place actors on stage, as in Ex. 40.

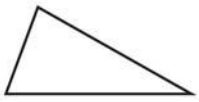
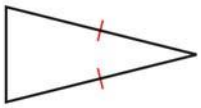
Key Vocabulary

- **triangle** scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- **interior angles**
- **exterior angles**
- **corollary to a theorem**

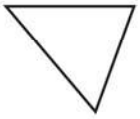
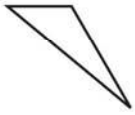
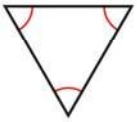
A **triangle** is a polygon with three sides. A triangle with vertices A, B, and C is called "triangle ABC" or "$\triangle ABC$."

KEY CONCEPT *For Your Notebook*

Classifying Triangles by Sides

Scalene Triangle	Isosceles Triangle	Equilateral Triangle
		
No congruent sides	At least 2 congruent sides	3 congruent sides

Classifying Triangles by Angles

Acute Triangle	Right Triangle	Obtuse Triangle	Equiangular Triangle
			
3 acute angles	1 right angle	1 obtuse angle	3 congruent angles

READ VOCABULARY

Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

EXAMPLE 1 Classify triangles by sides and by angles

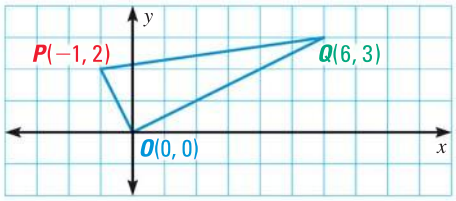
SUPPORT BEAMS Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.

Solution

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are 55°, 55°, and 70°. It is an acute isosceles triangle.

EXAMPLE 2 Classify a triangle in a coordinate plane

Classify $\triangle PQO$ by its sides. Then determine if the triangle is a right triangle.



Solution

STEP 1 **Use** the distance formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-1) - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-1))^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

STEP 2 **Check** for right angles. The slope of $\overline{OP}$ is $\frac{2 - 0}{-1 - 0} = -2$. The slope of $\overline{OQ}$ is $\frac{3 - 0}{6 - 0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$, so $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

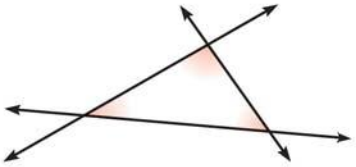
▶ Therefore, $\triangle PQO$ is a right scalene triangle.

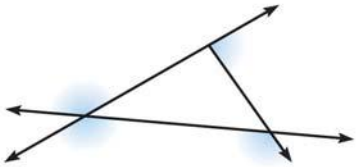
GUIDED PRACTICE for Examples 1 and 2

1. Draw an obtuse isosceles triangle and an acute scalene triangle.
2. Triangle ABC has the vertices $A(0, 0)$, $B(3, 3)$, and $C(-3, 3)$. Classify it by its sides. Then determine if it is a right triangle.

ANGLES When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

READ DIAGRAMS
Each vertex has a *pair* of congruent exterior angles. However, it is common to show only *one* exterior angle at each vertex.


interior angles

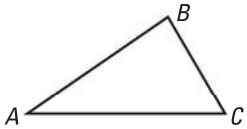

exterior angles

THEOREM For Your Notebook

THEOREM 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

Proof: p. 219; Ex. 53, p. 224



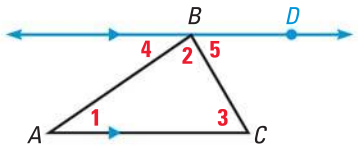
$m\angle A + m\angle B + m\angle C = 180°$

AUXILIARY LINES To prove certain theorems, you may need to add a line, a segment, or a ray to a given diagram. An *auxiliary* line is used in the proof of the Triangle Sum Theorem.

PROOF Triangle Sum Theorem

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Plan for Proof

a. Draw an auxiliary line through B and parallel to $\overline{AC}$.

b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180°$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

Plan in Action

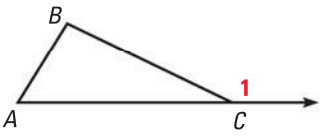
	STATEMENTS	REASONS
a.	**1.** Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	**1.** Parallel Postulate
b.	**2.** $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	**2.** Angle Addition Postulate and definition of straight angle
	3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	**3.** Alternate Interior Angles Theorem
	4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	**4.** Definition of congruent angles
c.	**5.** $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	**5.** Substitution Property of Equality

THEOREM *For Your Notebook*

THEOREM 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

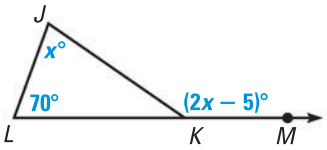
Proof: Ex. 50, p. 223



$m\angle 1 = m\angle A + m\angle B$

EXAMPLE 3 Find an angle measure

ALGEBRA Find $m\angle JKM$.



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$(2x - 5)° = 70° + x°$ Apply the Exterior Angle Theorem.

$x = 75$ Solve for x.

STEP 2 **Substitute** 75 for x in $2x - 5$ to find $m\angle JKM$.

$2x - 5 = 2 \cdot 75 - 5 = 145$

▶ The measure of $\angle JKM$ is 145°.

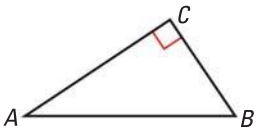
A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY *For Your Notebook*

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

Proof: Ex. 48, p. 223



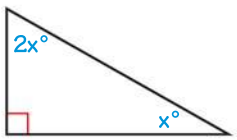
$m\angle A + m\angle B = 90°$

EXAMPLE 4 Find angle measures from a verbal description

ARCHITECTURE The tiled staircase shown forms a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.

Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.



Use the corollary to set up and solve an equation.

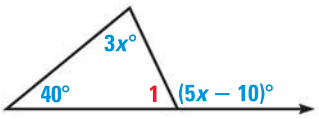
$x° + 2x° = 90°$ Corollary to the Triangle Sum Theorem

$x = 30$ Solve for x.

▶ So, the measures of the acute angles are $30°$ and $2(30°) = 60°$.

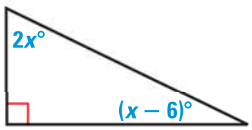
GUIDED PRACTICE for Examples 3 and 4

3. Find the measure of $\angle 1$ in the diagram shown.



4. Find the measure of each interior angle of $\triangle ABC$, where $m\angle A = x°$, $m\angle B = 2x°$, and $m\angle C = 3x°$.

5. Find the measures of the acute angles of the right triangle in the diagram shown.



6. In Example 4, what is the measure of the obtuse angle formed between the staircase and a segment extending from the horizontal leg?

4.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 15, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 7, 20, 31, 43, and 51

SKILL PRACTICE

VOCABULARY **Match the triangle description with the most specific name.**

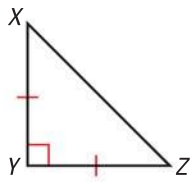
1. Angle measures: 30°, 60°, 90°	**A.** Isosceles
2. Side lengths: 2 cm, 2 cm, 2 cm	**B.** Scalene
3. Angle measures: 60°, 60°, 60°	**C.** Right
4. Side lengths: 6 m, 3 m, 6 m	**D.** Obtuse
5. Side lengths: 5 ft, 7 ft, 9 ft	**E.** Equilateral
6. Angle measures: 20°, 125°, 35°	**F.** Equiangular

7. ★ **WRITING** Can a right triangle also be obtuse? *Explain* why or why not.

EXAMPLE 1 on p. 217 for Exs. 8–10

CLASSIFYING TRIANGLES **Copy the triangle and measure its angles. Classify the triangle by its sides and by its angles.**

8.



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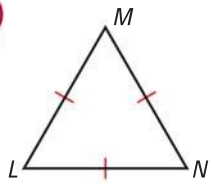

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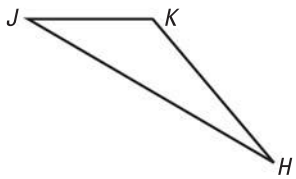


EXAMPLE 2 on p. 218 for Exs. 11–13

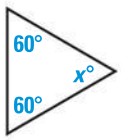
COORDINATE PLANE **A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle.**

11. $A(2, 3)$, $B(6, 3)$, $C(2, 7)$ **12.** $A(3, 3)$, $B(6, 9)$, $C(6, -3)$ **13.** $A(1, 9)$, $B(4, 8)$, $C(2, 5)$

EXAMPLE 3 on p. 219 for Exs. 14–19

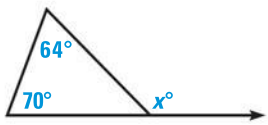
FINDING ANGLE MEASURES **Find the value of x. Then classify the triangle by its angles.**

14.



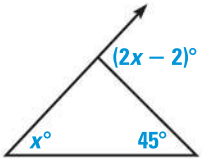
15.

16.



ALGEBRA **Find the measure of the exterior angle shown.**

17.



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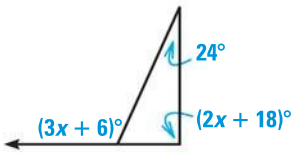

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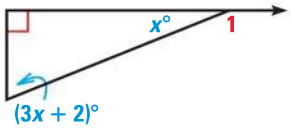
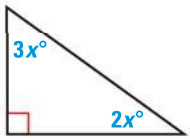


EXAMPLE 4 on p. 220 for Ex. 20

20. ★ **SHORT RESPONSE** *Explain* how to use the Corollary to the Triangle Sum Theorem to find the measure of each angle.

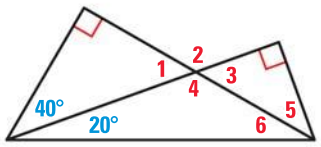


ANGLE RELATIONSHIPS **Find the measure of the numbered angle.**

21. $\angle 1$ **22.** $\angle 2$

23. $\angle 3$ **24.** $\angle 4$

25. $\angle 5$ **26.** $\angle 6$



27. **ALGEBRA** In $\triangle PQR$, $\angle P \cong \angle R$ and the measure of $\angle Q$ is twice the measure of $\angle R$. Find the measure of each angle.

28. **ALGEBRA** In $\triangle EFG$, $m\angle F = 3(m\angle G)$, and $m\angle E = m\angle F - 30°$. Find the measure of each angle.

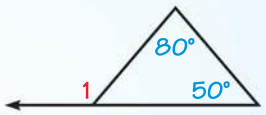
ERROR ANALYSIS **In Exercises 29 and 30, *describe* and correct the error.**

29.

All equilateral triangles are also isosceles. So, if $\triangle ABC$ is isosceles, then it is equilateral as well.

30.

$m\angle 1 + 80° + 50° = 180°$



31. ★ **MULTIPLE CHOICE** Which of the following is not possible?

(A) An acute scalene triangle

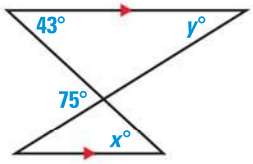
(B) A triangle with two acute exterior angles

(C) An obtuse isosceles triangle

(D) An equiangular acute triangle

ALGEBRA **In Exercises 32–37, find the values of x and y.**

32.



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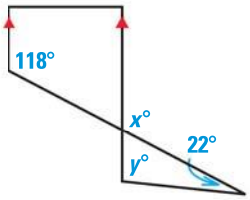

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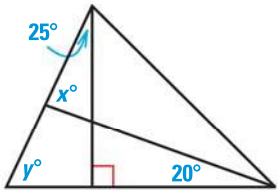

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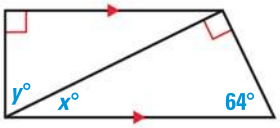

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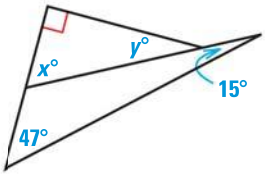

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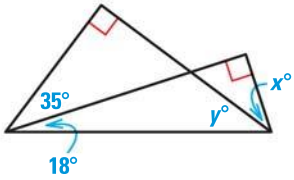


38. **VISUALIZATION** Is there an angle measure that is so small that any triangle with that angle measure will be an obtuse triangle? *Explain*.

39. **CHALLENGE** Suppose you have the equations $y = ax + b$, $y = cx + d$, and $y = ex + f$.

a. When will these three lines form a triangle?

b. Let $c = 1$, $d = 2$, $e = 4$, and $f = -7$. Find values of a and b so that no triangle is formed by the three equations.

c. Draw the triangle formed when $a = \frac{4}{3}$, $b = \frac{1}{3}$, $c = -\frac{4}{3}$, $d = \frac{41}{3}$, $e = 0$, and $f = -1$. Then classify the triangle by its sides.

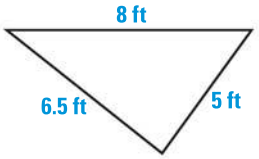
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 1 on p. 217 for Ex. 40

40. THEATER Three people are standing on a stage. The distances between the three people are shown in the diagram. Classify the triangle formed by its sides. Then copy the triangle, measure the angles, and classify the triangle by its angles.



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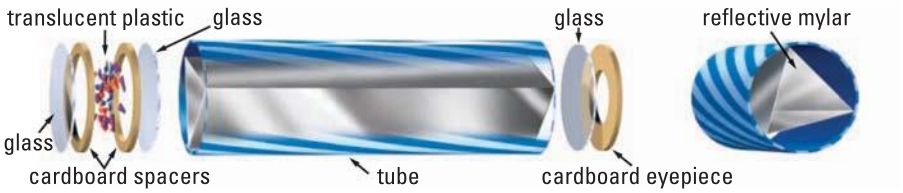
41. KALEIDOSCOPES You are making a kaleidoscope. The directions state that you are to arrange three pieces of reflective mylar in an equilateral and equiangular triangle. You must cut three strips from a piece of mylar 6 inches wide. What are the side lengths of the triangle used to form the kaleidoscope? What are the measures of the angles? *Explain*.



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42. SCULPTURE You are bending a strip of metal into an isosceles triangle for a sculpture. The strip of metal is 20 inches long. The first bend is made 6 inches from one end. *Describe* two ways you could complete the triangle.

43. ★ MULTIPLE CHOICE Which inequality describes the possible measures of an angle of a triangle?

Ⓐ $0° \leq x° \leq 180°$ Ⓑ $0° \leq x° < 180°$ Ⓒ $0° < x° < 180°$ Ⓓ $0° < x° \leq 180°$

SLING CHAIRS **The brace of a sling chair forms a triangle with the seat and legs of the chair. Suppose $m\angle 2 = 50°$ and $m\angle 3 = 65°$.**

44. Find $m\angle 6$.

45. Find $m\angle 5$.

46. Find $m\angle 1$.

47. Find $m\angle 4$.

48. PROOF Prove the Corollary to the Triangle Sum Theorem on page 220.

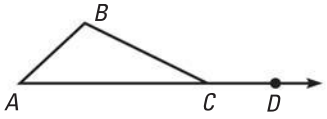
49. MULTI-STEP PROBLEM The measures of the angles of a triangle are $(2\sqrt{2x}°)$, $(5\sqrt{2x}°)$, and $(2\sqrt{2x}°)$.

a. Write an equation to show the relationship of the angles.

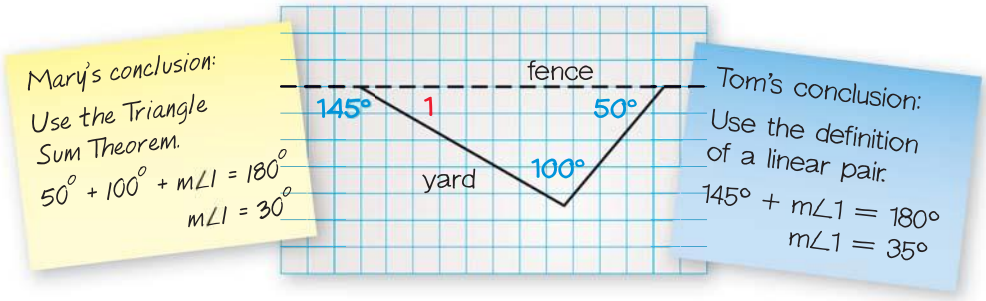
b. Find the measure of each angle.

c. Classify the triangle by its angles.

50. PROVING THEOREM 4.2 Prove the Exterior Angle Theorem. (*Hint:* Find two equations involving $m\angle ACB$.)



51. ★ **EXTENDED RESPONSE** The figure below shows an initial plan for a triangular flower bed that Mary and Tom plan to build along a fence. They are discussing what the measure of $\angle 1$ should be.



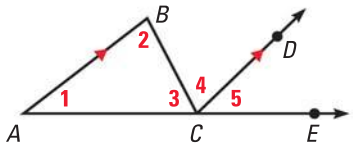
Did Mary and Tom both reason correctly? If not, who made a mistake and what mistake was made? If they did both reason correctly, what can you conclude about their initial plan? *Explain.*

52. **ALGEBRA** $\triangle ABC$ is isosceles. $AB = x$ and $BC = 2x - 4$.

a. Find two possible values for x if the perimeter of $\triangle ABC$ is 32.

b. How many possible values are there for x if the perimeter of $\triangle ABC$ is 12?

53. **CHALLENGE** Use the diagram to write a proof of the Triangle Sum Theorem. Your proof should be different than the proof of the Triangle Sum Theorem on page 219.



MIXED REVIEW

PREVIEW
Prepare for Lesson 4.2 in Exs. 57–59.

$\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$. *(p. 35)*

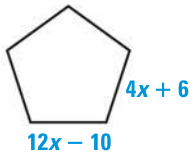
54. $m\angle A = (3x + 16)°$, $m\angle B = (4x - 3)°$

55. $m\angle A = (4x - 2)°$, $m\angle B = (7x + 4)°$

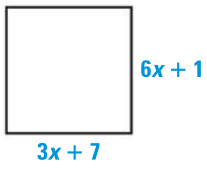
56. $m\angle A = (3x + 4)°$, $m\angle B = (2x + 6)°$

Each figure is a regular polygon. Find the value of x. *(p. 42)*

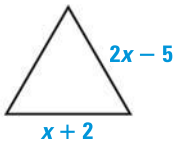
57.



58.



59.



60. Use the Symmetric Property of Congruence to complete the statement: If ? ≅ ?, then $\angle DEF \cong \angle PQR$. *(p. 112)*

Use the diagram at the right. *(p. 124)*

61. If $m\angle 1 = 127°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

62. If $m\angle 4 = 170°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

63. If $m\angle 3 = 54°$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.